


Permutation & Combination

Date Planned : __ / __ / __	Daily Tutorial Sheet – 12	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level - 3 	Exact Duration : _____

- 216.** A round robin chess tournament had $2n$ participants from two countries, n from each country. These were no two players with the same number of points at the end. Prove that there was at least one player who scored at least as many points against his compatriots as against the players of the other country. (In chess, a player gets one point for a win and one half a point for a draw).
- 217.** A magic square is a square matrix with non-negative integer entries in which all row sums and column sums are equal. Let $H_3(r)$ be the number of magic squares of size 3×3 in which each row and column have sum r . Prove that $H_3(r) = \binom{r+4}{4} + \binom{r+3}{4} + \binom{r+2}{4}$ where $H_3(r)$ is the number of 3×3 magic squares of line sum r .
- 218.** Give a combinational proof for:
- $$\sum \binom{n}{a_1, a_2, a_3} = 3^n$$
- 219.** Let $A = \{1, 2, 3, \dots, 100\}$ and B be a subset of A having 53 elements. Show that B has two distinct elements x and y whose sum is divisible by 11.
- 220.** Show that in any set of 20 distinct integers chosen from the set $\{1, 4, 7, \dots, 100\}$ there will always be two distinct integers whose sum is 104.
- 221.** I have six friends and during a certain vacation I met them during several dinners. I found that I dined with all the six exactly on one day, with every five of them on 2 days; with every four of them on three days; with every three of them on four days, with every two of them on five days. Further every friend was present and absent at 7 dinners each. How many dinners did I have alone?
(Hint: "I only know does not qualify as a legitimate answer.")